LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH SEMESTER – **APRIL 2023**

UST 4501 – ESTIMATION THEORY

Date: 02-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Define the following**
a)	Statistic
b)	Consistent estimator
c)	Completeness
d)	Bayesian estimate
e)	Confidence interval
2.	**Fill in the blanks**
a)	The expected value of difference between the true value of the parameter and estimator is called _______
b)	Factorization theorem is used to find __________statistic
c)	Lehmann-Scheffe theorem is used to obtain uniformly minimum ________ unbiased estimator.
d)	The invariance property is possessed by the maximum __________ estimator.
e)	The Bayes' estimator is _______ when the loss function is absolute error
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Match the following**
a)	Efficient estimator $\{\psi'(\theta)\}^2 / I_X(\theta)$
b)	Incomplete family Posterior distribution
c)	C-R lower bound Minimum chi-square
d)	Method of estimation Ratio of variances
e)	Bayes' estimation $\{N(0,\sigma^2), \sigma^2 > 0\}$
4.	**True or False**
a)	If X_1,X_2 is a random sample of size 2 from $B(1,\theta)$, $0 < \theta < 1$, then X_1+X_2 is sufficient for θ.
b)	If a minimum variance bound estimator exists then it is essentially unique.
c)	Maximum likelihood estimator is unique.
d)	An estimator which is asymptotically unbiased should be necessarily unbiased.
e)	Bayes' estimator is not unique.
	SECTION B - K3 (CO2)
	Answer any TWO of the following **(2 x 10 = 20)**
5.	State and prove Neyman-Fisher Factorization theorem.
6.	Show that the n^{th} order statistic is consistent for θ if $X_1,X_2,\ldots,X_n$ is a random sample from $U(0,\theta)$, $\theta > 0$.
7.	Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from $f(x;\theta) = \exp[-(x-\theta)]$, $x \geq \theta$,

	0 , otherwise. Find UMVUE of θ.
8.	Write the procedure for constructing the confidence interval for ratio of variances.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 10 = 20)**
9.	State and prove the following: (i)Rao-Blackwell theorem and (ii)Lehmann-Scheffe theorem. (5+5)
10.	Explain the following methods of estimation: (i)Maximum Likelihood (ii) Moments (iii)Minimum chi-square and (iv)Least squares. (3+3+2+2).
11.	Derive the Cramer – Rao inequality.
12.	Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from N(θ,1), $\theta \in$ R. Find the sufficient statistic and examine if it is complete.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 20 = 20)**
13.	(a)State and prove Chapman Robbins' inequality. (b) Let $X_1, X_2, \ldots, X_n$ be a random sample from N(μ, σ^2) . Find the sufficient statistic for μ when σ^2 is known and σ^2 when μ is known . (10+10)
14.	(a) Establish the invariance property of M.L.E. (b)Let $X_1, X_2, \ldots, X_n$ be a random sample from N(θ, 1) , $\theta \in$ R. Find Fisher's information contained in the sample. (c) Explain Loss and Risk functions. (5+10+5)
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
15.	(a)Let $X_1, X_2, \ldots, X_n$ be a random sample from B(1,θ) , $0 < \theta < 1$ and θ follows beta distribution of first kind with parameters α and β. Find Bayes' estimator of θ with respect to squared error loss function . (b)Narrate the construction of confidence interval for difference between means if the sampling is done from two normal populations. (10+10)
16.	(a) Establish with an example that M.L.E. is not consistent. (b) Prove with an example that M.L.E. is not sufficient . (14+6)

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